

## MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

# **Collisions involving antiparticles**

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Recent advances and future prospects in the study of atomic and molecular collisions using monoenergetic beams of positrons, positronium and antiprotons are considered as well as current efforts towards the synthesis of cold antihydrogen.

> Keywords: positronium formation; positron impact ionization; positron annihilation; positronium scattering; antiprotons; atomcule; antihydrogen

## 1. Introduction

The usage of beams of antiparticles with speeds of the order of those of atomic electrons is contributing to the unravelling of atomic physics phenomena (through the selective variation of the projectile mass and charge sign) as well as illuminating specific interactions and processes such as exchange and annihilation. Currently, considerable effort is also being directed towards the production of cold (less than kT) antiparticles, spurred by the prospects of investigations with low and/or well-defined energy, appropriate, for example, as indicated below, to the study of near-threshold and resonant phenomena or the production of cold antihydrogen for spectroscopic studies. In this paper we review some of the recent progress in these fields and consider future prospects. The interested reader is also referred to recent reviews by Raith (1998) and Laricchia & Charlton (1998).

## 2. Positrons

In most laboratories, radioisotopes are employed as the source of fast positrons  $(\beta^+)$ . A fraction of these, which are emitted with energies of the order of 100 keV, are slowed down to a few eV in a moderator that often consists of a metal or solid rare gas before being emitted into the vacuum. Electromagnetic fields are then used to confine and transport the slow positrons (away from the zone of high background near the source) to the experimentation region where they may be detected by charged-particle or gamma-ray detectors.

While the basic principle of producing a slow positron beam has remained unchanged over the past 20–30 years, advances in the understanding of the physics of positron moderation have resulted in an increase in moderation efficiencies of over  $10^5$  (see, for example, Schultz & Lynn 1988). This, coupled to improvements in source quality and activity, has resulted in intensities of up to  $10^7 e^+ s^{-1}$  for radioisotopebased beams (Greaves & Surko 1996). Beam strength of up to  $10^{10} e^+ s^{-1}$  are achievable at accelerators either through the production of electron–positron pairs via bremsstrahlung (Howell *et al.* 1982; Hulett *et al.* 1991) or through the production of intense (short-lived) isotopes (K. F. Canter, personal communication).

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Figure 1. Total cross-sections for positrons and electrons scattering from atomic and molecular hydrogen. The positronium formation thresholds are indicated by arrows. From Zhou *et al.* (1997).

While measurements of the total cross-section of positrons from simple atoms and molecules were among the first to be performed with monoenergetic beams, the experimental determination of the total cross-section from atomic hydrogen is a recent achievement, accomplished in a cooled aluminium cell characterized by a surface with a low recombination coefficient (Zhou *et al.* 1997). The results are shown in figure 1, where they are compared with those for molecular hydrogen and corresponding measurements with electrons. As illustrated in figure 1, often a minimum occurs at low energies in positron total cross-sections due to the partial cancellation of the static (repulsive) and polarization (attractive) interactions. In the case of electrons, these interactions are both attractive and they reinforce the scattering probability. It can also be noted that excellent agreement exists in the case of atomic hydrogen between recent theories and the experiment, except at the lowest energies where forward scattering errors are difficult to eliminate, partly due to the intrinsically poor beam energy resolution from typical moderators.

While studies of positron impact excitation remain sporadic primarily due to insufficient signal levels, data are available for elastic scattering from the inert atoms (both

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integral and differential) and some molecules, even though, in the latter case, the energy resolution has been in the main insufficient to discriminate against rotational and vibrational excitations (for reviews of these topics see, for example, Charlton & Laricchia 1990; Laricchia & Charlton 1998). However, a recent theoretical investigation (accompanied by qualitative experimental confirmation) has found that, for the symmetric stretching mode, the vibrational excitations of  $CO_2$  by electron impact are larger by two to three orders of magnitude than for positrons below 6 eV (Kimura *et al.* 1998).

Studies on positronium formation and direct ionization have been extensive in the past decade and now comprise integral studies with an atomic hydrogen target and, for other targets, near-threshold and differential investigations as well as multiple ionization (Kara *et al.* 1997, and references therein), including transfer ionization (Falke *et al.* 1995*a*, *b*, 1997; Bluhme *et al.* 1998).

Regarding positronium formation, a degree of consensus has been reached among experimental determinations of the integrated cross-section for atomic hydrogen (Zhou *et al.* 1997; Kara *et al.* 1998) and between these and theories (Kernoghan *et al.* 1996; Mitroy 1996; Higgins *et al.* 1990). Significant discrepancies exist between theory and experiment in the case of more complex targets (see, for example, Meyerhof & Laricchia 1997) and differential studies (see, for example, Garner *et al.* 1996; Finch *et al.* 1996; Falke *et al.* 1995*a*, 1997; McAlinden & Walters 1994). Although the formation of excited state positronium has been observed in positron–gas collisions (Laricchia *et al.* 1985), a systematic experimental study of the partitioning of the cross-section for Ps formation into its various quantum states to match corresponding calculations (Campbell *et al.* 1998*a*) is still lacking.

Again a degree of convergence has been reached with respect to measurements of the direct ionization of atomic hydrogen (Jones et al. 1993; Hofman et al. 1997) that is in agreement with most recent theories (see, for example, Kernoghan *et al.* 1996). As discussed further below, corresponding measurements with antiprotons have also been performed (Knudsen et al. 1995). In addition to atomic hydrogen, ionization data for all four  $(e^{\pm}, p^{\pm})$  projectiles are now available for the inert atoms (Paludan et al. 1997, and references therein) and some molecules (Jacobsen et al. 1995a, b; Moxom *et al.* 1996, and references therein). In general, at high velocities, the cross-sections for all four projectiles exhibit a similar energy dependence in accordance with the first Born approximation (FBA). At the lowest velocities, the cross-section for the positively charged projectiles are smaller than for their antiparticles, due to competition from electron capture and, in the case of the heavier projectiles, binding/antibinding effects (Knudsen & Reading 1992). At intermediate velocities, the heavier projectiles have higher cross-sections (mass effect) and, within each particle–antiparticle pair, the positively charged particles are more ionizing (charge effect). It should be noted, however, that while the latter statement appears to be valid for targets with low atomic numbers (Z), it is not generally true (Moxom et al. 1996; Laricchia 1995a, 1996). For targets with relatively large Z, the increasing static interaction between the light projectiles with the undistorted target results in progressively larger (smaller) impact parameters for the positron (electron) with associated deceleration (acceleration) of the projectile. The importance of such trajectory effects for the lighter particles is illustrated in figure 2, where the ratio between the single ionization cross-sections for the positive projec-



Figure 2. The ratios  $R^+ = Q_i^+(e^+)/Q_i^+(p^+)$  (diamonds and dotted lines) and  $R^- = Q_i^+(e^-)/Q_i^+(p^-)$  (circles and solid lines) are plotted versus projectile velocity. From Paludan *et al.* (1997).

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$$R^{+} = \frac{Q_{i}^{+}(e^{+})}{Q_{i}^{+}(p^{+})}$$

is plotted versus velocity for various targets and compared with that for the negatively charged projectiles

$$R^{-} = \frac{Q_{\rm i}^{+}({\rm e}^{-})}{Q_{\rm i}^{+}({\rm p}^{-})}$$

(Kara *et al.* 1997; Paludan *et al.* 1997). Here it can be seen that, while  $R^+ \sim R^-$  for H, H<sub>2</sub> and helium, as Z increases so does the difference between  $R^+$  and  $R^-$ , with  $R^+$  ( $R^-$ )  $\rightarrow 1$  progressively more slowly (quickly).

The behaviour of the positron-impact single-ionization cross-section for He and H<sub>2</sub> has been investigated for the first few eV above threshold (Ashley *et al.* 1996) and has been found to decrease more rapidly than for electrons as the threshold is approached but with an exponent (about 2) smaller than that predicted by the Wannier theory,  $Q_i^+(e^+) \propto (E')^{2.651}$ , E' being the incident energy minus that corresponding to the threshold. The results have been interpreted by Ihra *et al.* (1997) as arising from anharmonic terms in the three-particle potential around the Wannier configuration. In their work, such terms are actually found to be more important for positron- than



Figure 3. Triply differential cross-section (TDCS) in  $e^+(100 \text{ eV}) + H_2 \rightarrow e^+(0^\circ) + e^-(0^\circ, E_-) + H_2^+$ . Experimental data are from Kövér & Laricchia (1998), theory is from Berakdar (1998).

electron-impact ionization, implying correspondingly more stringent experimental requirements on beam quality for the former case.

Progress has also been made on angular- and energy-resolved studies of ionization that have recently shown, in the first triply differential measurement, the influence of electron capture to a low-lying continuum state of positronium (Kövér & Laricchia 1998; Berakdar 1998). This arises from the electron-positron correlations in which the positron and electron emerge from the collision unbound but with similar velocities. As shown in figure 3, the effect produces a cusp-like feature in the energy spectra of the ejected particles at half of the residual energy. The phenomenon was first observed in the ejected-electron energy spectra following the impact of heavy positively charged particles (see, for example, Rødbro & Andersen 1979; Schultz et al. 1991, and references therein). By contrast, negatively charged particles show the opposite so-called anticusp behaviour, since the particles are unlikely to emerge velocity-matched (see, for example, Pan et al. 1993; Golden et al. 1996; Schultz et al. 1991). Such studies, which have provided, in the case of electron impact (Lahmam-Bennani 1991), powerful insights into the physics of many-body correlated systems, are in their infancy in positron physics and much more work is anticipated in this area.

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Figure 4. Measurements of the temperature dependence of positron annihilation rates on noble gases compared to theory (lines). From Kurz *et al.* (1996).

Over the past 50 years, most information concerning annihilation has been derived from studies of  $\beta^+$  slowing down in dense media held at temperatures in the range from liquid helium to a few hundred degrees Celsius. In recent years, however, studies have advanced to systematic investigations of the lifetime of positrons of well-defined energy and under conditions of binary e<sup>+</sup>-molecule encounters (Surko *et al.* 1998). Data of this type are now available for *thermal* positrons annihilating from a vast range of atoms and molecules (Iwata *et al.* 1995), indicating a variation of the densitynormalized annihilation rate of over six orders of magnitude. In the case of the inert atoms, the annihilation rate has also been studied as a function of energy, albeit over a restricted range (Kurz *et al.* 1996). Figure 4 illustrates these results, which are found to agree, except at the highest energies, with corresponding calculations, displaying the decrease that had generally been expected with increasing positron energy (see, for example, Humberston 1979).

The vast variation of the annihilation probability with molecular species has recently been interpreted as being linked to the influence of virtual positronium (Laricchia & Wilkin 1997, 1998; Laricchia 1997). According to this interpretation, the virtual formation of positronium transiently traps the positron in the vicinity of the target high electron density and can give rise to pick-off annihilation (whereby the positron in the positronium atom annihilates with a target electron). As well as reproducing the large variation of the density-normalized annihilation rate (in broad accord with experiment), a semiempirical approach to the model has also led, as shown in figure 5, to the novel prediction of enhancements in the annihilation probability as energy thresholds are approached (Laricchia & Wilkin 1997,



Figure 5. Illustration of the qualitative variation of the ratio of the annihilation  $(Q_{ann})$  to elastic collision  $(Q_c)$  cross-sections plotted versus the incident positron energy for H, He, H<sub>2</sub> and C<sub>14</sub>H<sub>10</sub>. The peaks in each curve occur at the corresponding target thresholds for Ps formation. Also shown are the contributions to  $Q_{ann}$  arising from virtual Ps formation and in-flight annihilation. From Laricchia & Wilkin (1998).

1998). This prediction has been qualitatively confirmed by elaborate variational calculations (Van Reeth & Humberston 1998) and awaits experimental investigation.

## 3. Positronium

A comparatively recent development has been the realization of quasi-monoenergetic beams of positronium atoms (see, for example, Laricchia 1995b; Garner *et al.* 1996, and references therein). In practice, it is the triplet (or ortho-) ground state that makes up the beam, the singlet (or para-) ground state being untransportable by virtue of its lifetime (125 ps), being around three orders of magnitude shorter than for the orthostate. Although fast positronium may be produced readily by scattering fast positrons from matter, the best characterized method (resulting in a beam with a narrow energy spread) uses charge exchange in a gaseous target (Brown 1985; Laricchia *et al.* 1987). In this way the kinetic energy of the positronium atoms is tunable through that of the positron beam whose energy width sets the corresponding lower limit for the positronium formation in excited states, from positronium formation simultaneous to another inelastic process, etc.) may be controlled by appropriate choice of the neutralizing gas.

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Beams of this type have been used to measure directly total cross-sections of positronium scattering from argon (Zafar *et al.* 1996), helium and molecular hydrogen (Garner *et al.* 1996). As an example, the results for molecular hydrogen are shown in figure 6. The rapid rise with energy followed by a broad peak and a plateau is typical of the measured Ps cross-sections. Also shown are the calculations of Comi *et al.* (1983) for the sum of elastic and vibrational excitation cross-sections obtained within the context of multichannel scattering theory and those of Biswas & Ghosh (1996) performed by using the FBA. The latter theory neglects processes resulting in a change of the internal energy of the target, elastic scattering and even parity-state transitions of the positronium-excitation channel. Biswas & Adhikari (1998*a*) have calculated the electronic excitation of H<sub>2</sub> from the ground state to B<sup>1</sup> $\Sigma_{\rm u}^+$  and b<sup>3</sup> $\Sigma_{\rm u}^+$  by Ps impact within the framework of the FBA considering discrete excitation of Ps up to n = 6 and including break-up. Electron exchange is allowed through an extension to the Rudge approximation that takes into account the binding of the electron in the positronium atom.

As a further illustration of the considerable theoretical activity that has been stimulated by these novel Ps scattering experiments, a compilation of recent calculations for Ps–H is also given in figure 6. No experimental results are yet available for this fundamental but experimentally difficult collision system. Instead the results of Ps- $H_2$  scattering by Garner *et al.* (1996) are shown, divided by two for comparative purposes. The FBA calculations of McAlinden et al. (1996) include target inelastic processes. The calculations of Ray & Ghosh (1996, 1997) use a static exchange model (in which the ground states of both atoms are retained), while those of Sinha et al. (1997) employ a three-state Ps model close-coupling approximation (comprising elastic scattering and excitation to 2s and 2p states) with and without exchange. These results indicate that exchange is important below 50 eV. The calculations of Biswas & Adhikari (1998b) have been carried out using a three-state Ps closecoupling approximation including exchange plus higher states and the continuum evaluated via the FBA. Those of Sinha & Ghosh (1998) have investigated, within the close-coupling approximation, the effect of target inelastic collisions (neglecting exchange) and found them to be significant. Campbell et al. (1998b) have performed coupled pseudo-state calculations. As well as confirming an s-wave resonance first predicted by Drachman & Houston (1975), new resonances have been found in the singlet  $l \leq 5$  waves in the vicinity of 5 eV. These have been interpreted as arising from the binding of the positron to H<sup>-</sup>, an unstable system due to coupling to the decay into Ps excited states. Significant structure has also been found in the ortho-Ps to para-Ps conversion cross-section. It is felt that these very stimulating results will spur experimentalists further into investigating such a challenging system

Experimentally, the energy region below ca. 10 eV is made difficult by the rapidly decreasing transport and detection efficiencies of the slow Ps atoms and by the behaviour of the differential Ps formation cross-section that becomes progressively smaller and more isotropic as the incident positron energy is decreased. However, as illustrated by the results for H, this is a most interesting energy region where discrepancies between theories are particularly pronounced and of major concern, for example, in precision measurements of the o-Ps lifetime (Skalsey *et al.* 1998) and an understanding of the slowing down of positrons in dense media (Laricchia & Jacobsen 1985). In this respect, the investigation of specific Ps scattering channels

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Figure 6. Cross-sections for positronium scattering from molecular and atomic hydrogen. Experimental data (circles) are for molecular hydrogen.

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(and break-up in particular) is also highly desirable and should be achievable in the not-too-distant future. More long-term prospects include Ps scattering from surfaces (Canter 1983; Weber *et al.* 1988), spin-polarized studies (Laricchia 1995c), molecular positronium (see, for example, Varga *et al.* 1998) and Ps compounds (Schrader *et al.* 1996).

## 4. Antiprotons

For a decade or so, up to the closure of CERN's Low Energy Antiproton Ring (LEAR) in 1996, a number of atomic physics experiments with antiprotons were carried out. We briefly describe some of these efforts; more detailed reviews can be found elsewhere (Knudsen 1998; Eades & Hartmann 1999). The future for this type of work, outlined below, is now assured with the development of a new antiproton source at CERN, the Antiproton Decelerator (AD) (Maury 1997).

The first antiproton scattering experiments were those of Andersen *et al.* (1986) who investigated the double ionization of helium atoms. This work established that the differences between the cross-sections for this process, previously noted for electron and proton impact, were related to charge, rather than mass, as first suggested by McGuire (1982). Subsequently, there has been a great deal of experimental and theoretical effort expended in this area, and in related fields involving positron impact as described in § 2. As summarized there, much has been learnt concerning the dynamics of ionizing collisions through comparing particle and antiparticle scattering.

One of the major scattering achievements at LEAR was the study of antiprotonatomic-hydrogen ionization (Knudsen *et al.* 1995). Although the measured crosssections are described well by theory down to the lowest kinetic energy investigated (30 keV), and the theories themselves are in reasonable accord with one another, the latter is not true at lower energies. This situation has been summarized recently by Knudsen (1998), where the need for experimental work at the lower energies, hopefully to be undertaken at the AD, has been emphasized.

Antiprotonic helium (or 'atomcule') is formed when an antiproton is stopped in dense helium gas and replaces one of the electrons in a helium atom, giving rise to a state with a high angular momentum. Such states are metastable and can have a lifetime of a few microseconds (Iwasaki *et al.* 1991). The structure of the atomcule has been probed using laser spectroscopy, and several transitions, to states from which prompt annihilation ensued, have been observed (Morita *et al.* 1994; Yamazaki *et al.* 1997). Impressive agreement between the measured transition frequencies and those calculated using sophisticated three-body variational theories has been found (Kartavtsev 1996; Korobov & Shimamura 1997; Andersson *et al.* 1998).

For the AD physics programme, the two experimental strands mentioned above have formed a collaboration called ASACUSA (atomic spectroscopy and collisions using slow antiprotons) with the aim of performing a diverse series of investigations. These will be based around extracted beams, as at LEAR, but augmented by 100 keV beams from a radiofrequency decelerator and the use of an antiproton catching and cooling trap, similar to that described in § 5. This will allow lower energy collisions to be studied, including direct measurements on the formation of the atomcules before the states involved are affected by collisions with the surrounding medium. The antiproton–proton bound state, protonium, should also be amenable to experi-

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mentation. Spectroscopic studies of the atomcule will be continued and extended to hyperfine splittings.

## 5. Antihydrogen

Antihydrogen has recently been observed at CERN and Fermilab (Baur *et al.* 1996; Blanford et al. 1998), though only fleetingly and moving at relativistic speeds. In these experiments a rare interaction between an antiproton and an atomic nucleus resulted in the antiproton creating an electron-positron pair, but then emerging bound to the positron. The combination of the low rate of production and the high speed of the anti-atom poses a formidable challenge to the performance of physics experiments on antihydrogen produced by this method. It is envisaged that properties of antihydrogen such as the 1S-2S splitting, the ground-state hyperfine splitting and the Lamb shift would, through comparisons with the same quantities for ordinary hydrogen, be those most desirable for spectroscopic investigations (Charlton *et al.* 1994; Hänsch & Zimmermann 1993). Such comparisons are thought to offer tests of the CPT theorem and perhaps even the weak equivalence principle for antimatter (Holzscheiter & Charlton 1998; Eades 1995; Hughes 1993). To perform experiments on antihydrogen that might lead to meaningful tests of CPT, measurement precisions should be as high as possible. For instance, the 1S–2S transition, which has a natural linewidth of only around one part in  $10^{15}$  of its frequency, has often been discussed in this context. Currently, the best available measurements of this quantity for atomic hydrogen have fractional precisions in the  $10^{-12}$ - $10^{-13}$  range (Udem *et al.* 1997; Cesar et al. 1996), accuracies that have, in part, been achieved by working with cold atoms. This suggests that similar precisions will only be achieved for antihydrogen if cold anti-atoms can be produced.

To achieve this it is necessary to implement techniques to collect and cool positrons and antiprotons that have been developed in recent years. For positrons the most efficient technique to date uses a positron beam and buffer gas cooling, as pioneered by Surko and co-workers (Surko *et al.* 1989; Murphy & Surko 1992; Greaves *et al.* 1994), to collect the particles in a Penning–Malmberg trap. A suitably biased electrode structure, embedded in an axial magnetic field to provide radial confinement of the positrons, is employed and collisions with nitrogen gas provide the capture and cooling mechanisms. Positrons can be continuously accumulated in the trap and around  $10^8$  have been stored after 2–3 min. The lifetime of the positrons is *ca.* 1 min, but increases thirtyfold once the nitrogen is pumped out, whereupon they can be moved to another high-vacuum section of the apparatus for long-term storage and antihydrogen production (Surko *et al.* 1997).

Techniques to capture and cool antiprotons were first developed by Gabrielse and co-workers (Gabrielse *et al.* 1986, 1989) and have since been applied by others (Holzscheiter *et al.* 1996; Feng *et al.* 1997). The method relies on the dynamical capture of a small fraction of the pulsed output of LEAR (in future the AD) following the slowing down of the antiprotons as they pass through material on entering the trap. The antiprotons captured in this fashion are confined by the electric and magnetic field present in the trap and pass back and forth through a cold electron cloud to which they couple via the Coulomb interaction. The antiprotons sympathetically cool, and after a period of *ca.* 1 min reach thermal equilibrium with the

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electrons (and thus the surrounding trap). More than  $10^6$  antiprotons from a single LEAR pulse (Holzscheiter *et al.* 1996) have been cooled in this fashion.

It is envisaged that the cold clouds of antiparticles will be combined to form antihydrogen and some of the reactions that might allow this to be achieved can be summarized as follows:

$e^+ + \bar{p} \to \bar{H} + h\nu$	spontaneous radiative recombination,
$e^+ + \bar{p} + mh\nu \rightarrow \bar{H} + (m+1)h\nu$	stimulated recombination,
$e^+ + e^+ + \bar{p} \rightarrow \bar{H} + e^+$	three-body recombination,
$\mathrm{Ps} + \bar{\mathrm{p}} \rightarrow \bar{\mathrm{H}} + \mathrm{e}^-$	charge transfer from positronium.

Charge conjugate reactions of the spontaneous radiative (see, for example, Budker & Skrinsky 1978) and stimulated (Schramm *et al.* 1991; Yousif *et al.* 1991) recombination reactions have been observed, but in merged-beam, rather than trap-type, experiments. Hydrogen formation due to proton–positronium collisions has also been observed recently (Merrison *et al.* 1997). Although this latter reaction involved the use of a keV energy proton beam, the kinematic conditions corresponded to a collision involving low-energy (eV) positronium with cold antiprotons. Each of the reactions listed above has some advantages and disadvantages and research to obtain a more detailed understanding of their implementation in traps is ongoing. However, it seems likely that one or other of the positron–antiproton reactions will be the favoured method.

A schematic illustration of the apparatus of one of the antihydrogen experiments scheduled for operation at the AD (that of the ATHENA collaboration) is given in figure 7, where positron and antiproton traps similar to those described above are shown. Other important elements of the apparatus are the recombination trap, the magnetic gradient trap for the neutral antihydrogen and a detector. A description of the apparatus has been given by Holzscheiter *et al.* (1997), such that a brief overview will suffice here.

The antihydrogen detector consists of two sections. The first will be used to register the back-to-back 511 keV positron annihilation gamma rays, while the second, an array of silicon strip counters, will detect the shower of charged pions originating from the annihilation of the antiproton. The detector has position sensitivity that should enable the point of annihilation to be determined with millimetre-type accuracy.

The recombination trap is envisaged as being of the nested-wells type, as first suggested by Gabrielse *et al.* (1988). Here the oppositely charged particles are each held in embedded potential wells of appropriate polarity. It is hoped that this will allow the plasmas to be merged in a controlled manner to promote recombination. This trap, and the antiproton and positron traps described above, will be housed in the bore of a superconducting solenoid producing an axial magnetic field of around 3 T. However, this field will be reduced to *ca.* 0.5 T in the region of the recombination trap if the final element of the apparatus, the magnetic gradient neutral trap, is operated. The purpose of the latter (see, for example, Gomer *et al.* 1997) is to enable the trapping of a few antihydrogen atoms created with temperatures lower than the typical trap depth. Using currently available magnet technology this depth is expected to be less than or equal to 1 K for the geometry of the ATHENA apparatus. The well is created by a system of coils that together form a three-dimensional magnetic field minimum. The coils will be a Helmholtz pair aligned with the main

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Figure 7. Schematic illustration of the proposed ATHENA apparatus. From Holzscheiter et~al.~(1997).

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solenoid, but with their field opposed, to create a field minimum along the axis of the system, and two pairs of racetrack, or quadrupole, coils arranged along the solenoid axis to create the confining field in the other two dimensions.

The main aim of ATHENA is to perform precision spectroscopy on trapped antihydrogen atoms to make detailed comparisons with the same transitions in ordinary hydrogen. Clearly, much development work needs to be done to realize this ambitious goal. Nevertheless, as outlined here, most of the necessary steps have been demonstrated such that we can anticipate a bright future for antimatter research.

## 6. Conclusions

In recent years, there have been considerable advances in the understanding of the interactions of low-energy positrons, positronium and antiprotons with atomic and molecular matter. It is envisaged that the vigorous progress in experimental techniques will ensure that some of the more challenging collision systems and energy domains will become measurable in the foreseeable future.

It is a pleasure to thank all our co-workers and collaborators at UCL and elsewhere. We are grateful to the Engineering and Physical Science Research Council, the Royal Society and NATO for their continuing support of positron research at UCL. This article is dedicated to Professor P. G. Burke on the occasion of his retirement. His support of the field of atomic collisions with low-energy positrons has been of major benefit to its development into a flourishing area of UK physics.

## References

- Andersen, L. H., Hvelplund, P., Knudsen, H., Moller, S. P., Elsener, K., Rensfelt, K.-G. & Uggerhoj, E. 1986 Single and double ionization of helium by fast antiproton and proton impact. *Phys. Rev. Lett.* 57, 2147–2150.
- Andersson, S., Elander, N. & Yarevsky, E. 1998 A study of isotope effects in the antiprotonic helium system. J. Phys. B 31, 625–632.
- Ashley, P., Moxom, J. & Laricchia, G. 1996 Near-threshold ionization of He and H<sub>2</sub> by positron impact. *Phys. Rev. Lett.* 77, 1250–1253.
- Baur, G. (and 23 others) 1996 Production of antihydrogen. Phys. Lett. B 368, 251–258.
- Berakdar, J. 1998 Asymmetric formation of positronium continuum states following positronimpact ionization of H<sub>2</sub>. Phys. Rev. Lett. 81, 1393–1396.
- Biswas, P. K. & Adhikari, S. K. 1998a Ps impact excitation of hydrogen molecule to  $B^{1}\Sigma_{u}^{+}$  and  $b^{3}\Sigma_{u}^{+}$  states. J. Phys. Lett. B **31**, 315–320.
- Biswas, P. K. & Adhikari, S. K. 1998b Effect of electron exchange in Ps–H scattering. J. Phys. B **31**, 3147–3154.
- Biswas, P. K. & Ghosh, A. S. 1996 Excitation and ionization of the positronium atom in Ps-H<sub>2</sub> molecule scattering. *Phys. Lett.* A 29, 173–178.
- Blanford, G., Christian, D. C., Gollwitzer, K., Mandelkern, M., Munger, C. T., Schultz, J. & Zioulas, G. 1998 Observation of atomic antihydrogen. *Phys. Rev. Lett.* 80, 3037–3040.
- Bluhme, H., Knudsen, H., Merrison, J. P. & Poulsen, M. R. 1998 Strong suppression of the positronium channel in double ionization of noble gases by positron impact. *Phys. Rev. Lett.* 81, 73–76.
- Brown, B. L. 1985 Creation of a monoenergetic Ps beam in a gas. In *Positron annihilation* (ed. P. C. Jain, R. M. Singru & K. P. Gopinathan), pp. 212–221. Singapore: World Scientific.
- Budker, G. & Skrinsky, A. N. 1978 Electron cooling and new possibilities in elementary particle physics. Sov. Phys. Usp. 21, 277–296.

Campbell, C. P., McAlinden, M. T., MacDonald, F. R. G. S. & Walters, H. R. J. 1998a Positron collisions with one- and two-electron atoms. *Nucl. Instr. Meth.* B 143, 41–56.

- Campbell, C. P., McAlinden, M. T., MacDonald, F. R. G. S. & Walters, H. R. J. 1998b Scattering of positronium by atomic hydrogen. *Phys. Rev. Lett.* 80, 5097–5100.
- Canter, K. F. 1983 Low energy positron and positronium diffraction. In *Positron scattering in gases* (ed. J. W. Humberston & M. R. C. McDowell), pp. 219–225. New York: Plenum.
- Cesar, C. L., Fried, D., Killian, T., Polcyn, A., Sandberg, J., Yu, I. A., Greytak, T. J., Kleppner, D. & Doyle, J. M. 1996 Two-photon spectroscopy of trapped atomic hydrogen. *Phys. Rev. Lett.* 77, 255–258.
- Charlton, M. & Laricchia, G. 1990 Positron impact phenomena. J. Phys. B 23, 1047–1078.
- Charlton, M., Eades, J., Horváth, D., Hughes, R. J. & Zimmermann, C. 1994 Antihydrogen physics. Phys. Rep. 241, 65–117.
- Comi, M., Prosperi, G. M. & Zecca, A. 1983 Elastic and vibrational excitation cross-sections in Ps-H<sub>2</sub> scattering. *Nuovo Cimento* 2, 1347–1375.
- Drachman, R. J. & Houston, S. K. 1975 Positronium-hydrogen elastic scattering. Phys. Rev. A 12, 885–890.
- Eades, J. 1995 Testing fundamental symmetries with antihydrogen. Comments At. Mol. Phys. 31, 51–66.
- Eades, J. & Hartmann, F. J. 1999 Forty years of antiprotons. Rev. Mod. Phys. 71, 373-419.
- Falke, T., Raith, W., Weber, M. & Wesskamp, U. 1995a Differential Ps formation in positron– argon collisions. J. Phys. Lett. B 28, 505–509.
- Falke, T., Raith, W. & Weber, M. 1995b Observation of structure in differential transfer ionization of argon atoms by positrons. *Phys. Rev. Lett.* **75**, 3418–3421.
- Falke, T., Brandt, T., Kuhl, O., Raith, W. & Weber, M. 1997 Differential Ps formation and impact ionization cross-sections for positron scattering on Ar and Kr atoms. J. Phys. B 30, 3247–3256.
- Feng, X., Holzscheiter, M. H., Charlton, M., Hangst, J., King, N. S. P., Lewis, R. A., Rochet, J. & Yamazaki, Y. 1997 Capture and cooling of antiprotons. *Hyperfine Interactions* 109, 145–152.
- Finch, R. M., Kövér, A., Charlton, M. & Laricchia, G. 1996 Positron–argon elastic, Ps formation and ionizing collisions at 60°. J. Phys. Lett. B 29, 667–672.
- Gabrielse, G., Fei, X., Helmerson, K., Rolston, S. L., Tjoekler, R., Trainor, T. A., Kalinowsky, H., Haas, J. & Kells, W. 1986 First capture of antiprotons in a Penning trap: a kiloelectronvolt source. *Phys. Rev. Lett.* 57, 2504–2507.
- Gabrielse, G., Rolston, S. L., Haarsma, L. & Kells, W. 1988 Possible antihydrogen production using trapped plasmas. *Hyperfine Interactions* 44, 287–294.
- Gabrielse, G., Fei, X., Orozco, L. A., Tjoekler, R., Haas, J., Kalinowsky, H., Trainor, T. A. & Kells, W. 1989 Cooling and slowing of trapped antiprotons below 100 meV. *Phys. Rev. Lett.* 63, 1360–1363.
- Garner, A. J., Laricchia, G. & Ozen, A. 1996 Ps beam production and scattering from gaseous targets. J. Phys. B 29, 5961–5968.
- Golden, D. E., Xu, Z., Bernhard, J. & Mueller, D. W. 1996 Electron anticapture to the continuum in double-binary electron-atom collisions. J. Phys. B 29, 3741–3746.
- Gomer, V., Harms, O., Haubrich, D., Schadwinkel, H., Strauch, F., Ueberholz, B., aus der Wiesche, S. & Meschede, D. 1997 Magnetostatic traps for charged and neutral particles. *Hyperfine Interactions* 109, 281–292.
- Greaves, R. G. & Surko, C. M. 1996 Solid neon moderator for positron trapping experiments. Can. Jl Phys. 74, 445–448.
- Greaves, R. G., Tinkle, M. D. & Surko, C. M. 1994 Creation and uses of positron plasmas. *Phys. Plasmas* 1, 1439–1446.

- Hänsch, T. W. & Zimmermann, C. 1993 Laser spectroscopy of hydrogen and antihydrogen. *Hyperfine Interactions* 76, 47–57.
- Higgins, K., Burke, P. G. & Walters, H. R. J. 1990 Positron scattering by atomic hydrogen at intermediate energies. J. Phys. B 23, 1345–1357.
- Hofmann, A., Falke, T., Raith, W., Weber, M., Becker, D. P. & Lynn, K. G. 1997 Ionization of atomic hydrogen by positrons. J. Phys. B 30, 3297–3303.
- Holzscheiter, M. H. & Charlton, M. 1998 Ultra-low energy antihydrogen. Rep. Prog. Phys. 62, 1–60.
- Holzscheiter, M. H., Feng, X., Goldman, T., King, N. S. P., Lewis, R. A., Nieto, M. M. & Smith, G. A. 1996 Are antiprotons forever? *Phys. Lett.* A 214, 279–284.
- Holzscheiter, M. H. (and 53 others) 1997 Antihydrogen production and precision experiments. Nucl. Phys. B 56, 336–348.
- Howell, R. H., Alvarez, R. A. & Stanek, M. 1982 Production of slow positrons with a 100 MeV electron linac. Appl. Phys. Lett. 40, 751–752.
- Hulett, L. D., Donohue, D. L. & Lewis, T. A. 1991 A quadratic potential time-of-flight massspectrometer for use with slow positron ionization sources and other large-volume ion sources. *Rev. Sci. Instrum.* 62, 2131–2137.
- Hughes, R. J. 1993 The equivalence principle. Contemp. Phys. 34, 177–191.
- Humberston, J. W. 1979 Theoretical aspects of positron collisions in gases. Adv. At. Mol. Phys. 15, 101–133.
- Ihra, W., Macek, J. H., Mota-Furtado, F. & O'Mahony, P. F. 1997 Threshold law for positron impact ionization of atoms. *Phys. Rev. Lett.* 78, 4027–4030.
- Iwasaki, M. (and 12 others) 1991 Discovery of antiproton trapping by long-lived metastable states in liquid helium. Phys. Rev. Lett. 67, 1246–1249.
- Iwata, K., Greaves, R. G., Murphy, T. J., Tinkle, M. D. & Surko, C. M. 1995 Measurements of positron-annihilation rates on molecules. *Phys. Rev.* A 51, 473–487.
- Jacobsen, F. M., Frandsen, N. P., Knudsen, H. & Mikkelsen, U. 1995a Non-dissociative single ionization of molecular hydrogen by electron and positron impact. J. Phys. B 28, 4675–4689.
- Jacobsen, F. M., Frandsen, N. P., Knudsen, H., Mikkelsen, U. & Schrader, D. M. 1995b Single ionization of He, Ne and Ar by positron impact. J. Phys. B 28, 4691–4695.
- Jones, G. O., Charlton, M., Slevin, J., Laricchia, G., Kövér, A., Poulsen, M. R. & Chormaic, S. N. 1993 Positron impact ionization of atomic hydrogen. J. Phys. Lett. B 26, 483–488.
- Kara, V., Paludan, K., Moxom, J., Ashley, P. & Laricchia, G. 1997 Single and double ionization of Ne, Kr and Xe by positron impact. J. Phys. B 30, 3933–3949.
- Kara, V., Paludan, K., Moxom, J., Ashley, P. & Laricchia, G. 1998 Positron impact ionization of atoms. Nucl. Instrum. Meth. B 143, 94–99.
- Kartavtsev, O. I. 1996 Variational calculations of antiprotonic helium atoms. Russian Jl Nucl. Phys. 59, 1541–1550.
- Kernoghan, A. A., Robinson, D. J. R., McAlinden, M. T. & Walters, H. R. J. 1996 Positron scattering from atomic hydrogen. J. Phys. B 29, 2089–2102.
- Kimura, M., Takekawa, M., Itikawa, Y., Takaki, H. & Sueoka, O. 1998 Mode dependence in vibrational excitation of a CO<sub>2</sub> molecule by electron and positron impacts. *Phys. Rev. Lett.* 80, 3936–3939.
- Knudsen, H. 1998 Antiprotons colliding with atoms and molecules. In *Photonic, electronic and atomic collisions* (ed. F. Aumayr & H. P. Winter), pp. 453–464. Singapore: World Scientific.
- Knudsen, H. & Reading, J. F. 1992 Ionization of atoms by particle and antiparticle impact. *Phys. Rep.* 212, 107–222.
- Knudsen, H., Mikkelsen, U., Paludan, K., Kirsebom, K., Moller, S. P., Uggerhoj, E., Slevin, J., Charlton, M. & Morenzoni, E. 1995 Ionization of atomic hydrogen by 30–1000 keV antiprotons. *Phys. Rev. Lett.* **74**, 4627–4630.

- Kövér, A. & Laricchia, G. 1998 Triply differential study of positron impact ionization of H<sub>2</sub>. *Phys. Rev. Lett.* **80**, 5309–5312.
- Korobov, V. I. & Shimamura, I. 1997 Auger transition rates for metastable states of antiprotonic helium. Phys. Rev. A 56, 4587–4594.
- Kurz, C., Greaves, R. G. & Surko, C. M. 1996 Temperature-dependence of positron-annihilation rates in noble gases. *Phys. Rev. Lett.* 77, 2929–2932.
- Lahmam-Bennani, A. 1991 Recent developments and new trends in (e, 2e) and (e, 3e) studies. J. Phys. B 24, 2401–2442.
- Laricchia, G. 1995a Positron and positronium collisions. Nucl. Instrum. Meth. B 99, 363-367.
- Laricchia, G. 1995b Positronium beams and surfaces. In Positron spectroscopy of solids. Proc. Int. School of Physics, Enrico Fermi (ed. A. Dupasquier & A. P. Mills), vol. 125, pp. 401–418. Amsterdam: IOS.
- Laricchia, G. 1995c Studies with polarized positrons. In *Polarized electron-polarized photon physics* (ed. H. Kleinpoppen & W. R. Newell), pp. 169–176. New York: Plenum.
- Laricchia, G. 1996 Of atomic collisions with positrons and positronium. Hyperfine Interactions 100, 71–89.
- Laricchia, G. 1997 Of positron scattering and annihilation from molecules. Mater. Sci. Forum 255, 228–232.
- Laricchia, G. & Charlton, M. 1998 Atomic and molecular physics with positrons and positronium. In *Positron beams and their applications* (ed. P. G. Coleman). Singapore: World Scientific.
- Laricchia, G. & Jacobsen, F. M. 1985 Positron slowing down in molecular hydrogen: a Monte Carlo study. In *Positron (electron)-gas scattering* (ed. W. E. Kauppila, S. Stein & J. M. Wadehra), pp. 329–333. Singapore: World Scientific.
- Laricchia, G. & Wilkin, C. 1997 Semiempirical approach to positron annihilation in molecules. *Phys. Rev. Lett.* 79, 2241–2244.
- Laricchia, G. & Wilkin, C. 1998 On the annihilation of positrons in binary encounters with molecules. Nucl. Instrum. Meth. B 143, 135–139.
- Laricchia, G., Charlton, M., Clark, G. & Griffith, T. C. 1985 Excited state Ps formation in low-density gases. *Phys. Lett.* A 109, 97–100.
- Laricchia, G., Charlton, M., Davies, S. A., Beling, C. D. & Griffith, T. C. 1987 The production of collimated beams of o-Ps atoms using charge exchange in positron-gas collisions. J. Phys. Lett. B 20, 99–105.
- Laricchia, G., Zafar, N., Charlton, M. & Griffith, T. C. 1992 Recent advances in the characterization of a Ps beam. *Hyperfine Interactions* 73, 133–145.
- McAlinden, M. T. & Walters, H. R. J. 1994 Differential cross-sections for elastic scattering and Ps formation for positron collisions with Ne, Ar, Kr and Xe. *Hyperfine Interactions* 84, 407–418.
- McAlinden, M. T., MacDonald, F. R. G. S. & Walters, H. R. J. 1996 Positronium-atom scattering. Can. Jl Phys. 74, 434–444.
- McGuire, J. H. 1982 Double ionization of helium by protons and electrons at high velocities. *Phys. Rev. Lett.* 49, 1153–1157.
- Maury, S. 1997 The antiproton decelerator: AD. Hyperfine Interactions 109, 43-52.
- Merrison, J. P., Bluhme, H., Chevallier, J., Deutch, B. I., Hvelplund, P., Jorgensen, L. V., Knudsen, H., Poulsen, M. R. & Charlton, M. 1997 Hydrogen formation by proton impact on positronium. *Phys. Rev. Lett.* **78**, 2728–2731.
- Meyerhof, W. E. & Laricchia, G. 1997 Threshold and channel-coupling effects in positron elastic scattering from noble gases. J. Phys. B **30**, 2221–2238.
- Mitroy, J. 1996 An  $L^2$  calculation of positron-hydrogen scattering at intermediate energies. J. Phys. Lett. B 29, 263–269.

- Morita, N. (and 16 others) 1994 First observation of laser-induced resonant annihilation in metastable antiprotonic helium atoms. *Phys. Rev. Lett.* **72**, 1180–1183.
- Moxom, J., Ashley, P. & Laricchia, G. 1996 Single ionization by positron impact. *Can. Jl Phys.* **74**, 367–372.
- Murphy, T. J. & Surko, C. M. 1992 Positron trapping in an electrostatic well by inelastic collisions with nitrogen molecules. *Phys. Rev.* A 46, 5696–5705.
- Paludan, K. (and 10 others) 1997 Ionization of rare gases by particle–antiparticle impact. J. Phys. Lett. B 30, 581–587.
- Pan, G. Y., Hvelplund, P., Knudsen, H., Yamazaki, Y., Brauner, M. & Briggs, J. S. 1993 Electron correlation in the continuum: the e-2e process for small relative momenta of the outgoing electrons. *Phys. Rev.* A 47, 1531–1534.
- Raith, W. 1998 Electron and positron scattering by atoms. In *Photonic, electronic and atomic collisions* (ed. F. Aumayr & H. Winter), pp. 341–360. Singapore: World Scientific.
- Ray, H. & Ghosh, A. S. 1996 Ps–H scattering using the static exchange model. J. Phys. B 29, 5505–5511 (1997 Corrigendum J. Phys. B 30, 3745–3746).
- Rødbro, M. & Andersen, F. D. 1979 Charge transfer to the continuum for 15 to 1500 keV H<sup>+</sup> in He, Ne, Ar and H<sub>2</sub> gases under single-collision conditions. J. Phys. B 12, 2883–2903.
- Schrader, D. M., Hulett, L. D., Xu, J., Laricchia, G., Lewis, T. A. & Horsky, T. N. 1996 The ORELA positron-scattering program. *Can. Jl Phys.* 74, 497–500.
- Schramm, U., Berger, J., Grieser, M., Habs, D., Jaeschke, E., Kilgus, G., Schwalm, D., Wolf, A., Neumann, R. & Schuch, R. 1991 Observation of laser-induced recombination in merged electron and proton beams. *Phys. Rev. Lett.* 67, 22–25.
- Schultz, P. J. & Lynn, K. G. 1988 Interaction of positron beams with surfaces, thin films and interfaces. *Rev. Mod. Phys.* 60, 701–779.
- Schultz, D. R., Olson, R. E. & Reinhold, C. O. 1991 Recent advances in the comparison of matter-atom and antimatter-atom collisions. J. Phys. B 24, 521–558.
- Sinha, P. K. & Ghosh, A. S. 1998 Ps–H scattering using the six-state close-coupling approximation. Phys. Rev. A 58, 242–249.
- Sinha, P. K., Chaudhury, P. & Ghosh, A. S. 1997 Ps–H scattering using the three-state Ps close-coupling approximation. J. Phys. B **30**, 4643–4652.
- Skalsey, M., Engbrecht, J. J., Bithell, R. K., Vallery, R. S. & Gidley, D. W. 1998 Thermalization of positronium in gases. *Phys. Rev. Lett.* 80, 3727–3730.
- Surko, C. M., Leventhal, M. & Passner, A. 1989 Positron plasma in the laboratory. *Phys. Rev. Lett.* 62, 901–904.
- Surko, C. M., Greaves, R. G. & Charlton, M. 1997 Stored positrons for antihydrogen production. *Hyperfine Interactions* 109, 181–188.
- Surko, C. M., Iwata, K., Greaves, R. G., Kurz, C. & Gilbert, S. J. 1998 Atomic and molecular physics using positrons in a Penning trap. In *Photonic, electronic and atomic collisions* (ed. F. Aumayr & H. Winter), pp. 383–392. Singapore: World Scientific.
- Udem, Th., Huber, A., Gross, B., Reichert, J., Predevelli, M., Weitz, M. & Hänsch, T. W. 1997 Phase coherent measurement of the hydrogen 1S–2S transition frequency with an optical frequency interval divider chain. *Phys. Rev. Lett.* **79**, 2646–2649.
- Van Reeth, P. & Humberston, J. W. 1998 The energy dependence of the annihilation rate in positron-atom scattering. J. Phys. Lett. B 30, 231–238.
- Varga, K., Usukura, J. & Suzuki, Y. 1998 Second bound state of the positronium molecule and biexcitons. *Phys. Rev. Lett.* 80, 1876–1879.
- Weber, M. H., Tang, S., Berko, S., Brown, B. L., Canter, K. F., Lynn, K. G., Mills, A. P., Roellig, L. O. & Viescas, A. J. 1988 Observation of positronium specular reflection from LiF. *Phys. Rev. Lett.* **61**, 2542–2545.

- Yamazaki, T. (and 20 others) 1997 High precision structural studies of the antiprotonic helium atom p<sup>4</sup>He<sup>+</sup> by observing laser resonances with  $\Delta u = \Delta(n l 1) = 2$ . *Phys. Rev.* A 55, R3295–R3298.
- Yousif, F. B., Van der Donk, P., Kucherovsky, Z., Reis, J., Brannen, E., Mitchell, J. B. A. & Morgan, T. J. 1991 Experimental observation of laser-stimulated radiative recombination. *Phys. Rev. Lett.* 67, 26–29.
- Zafar, N., Laricchia, G., Charlton, M. & Garner, A. 1996 Positronium-argon scattering. Phys. Rev. Lett. 4, 1595–1598.
- Zhou, S., Li, H., Kauppila, W. E., Kwan, C. K. & Stein, T. S. 1997 Measurement of the total and Ps formation cross-sections for positrons and electrons from hydrogen atoms and molecules. *Phys. Rev.* A 55, 361–368.

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